Weyl Semimetals

Beyond Band Insulators: Topology of Semi-metals and Interacting Phases

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(Dated: January 4, 2013)

Journal Club, 09/08/2013

Semimetals: basic questions

- Is it possible for a gapless system to have a defining topological feature?
- How can one distinguish phases of a system with gapless degrees of freedom?
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- Weyl semimetal, as an example

Brief review: Dirac equation

- Dirac equation for a spin-1/2 particle with mass m

$$\left(i\gamma^{\mu}\partial_{\mu} - m\right)\psi = 0$$

$$\left\{\gamma^{\mu}, \gamma^{\nu}\right\} = 2\eta^{\mu\nu}$$

Dirac algebra

- Weyl representation:

$$\gamma^0 = \tau_1 \otimes I = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

$$\gamma^i = i\tau_2 \otimes \sigma_i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}$$

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = i\left(\tau_1 \otimes I\right)\left(i\tau_2 \otimes \sigma_1\right)\left(i\tau_2 \otimes \sigma_2\right)\left(i\tau_2 \otimes \sigma_3\right) = \begin{pmatrix} -I & 0 \\ 0 & I \end{pmatrix}$$

- γ^5 is diagonal (Dirac representation: γ^0 is diagonal)

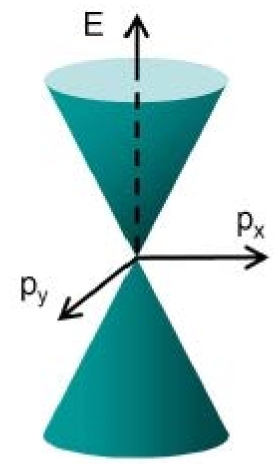
Brief review: Dirac equation

- In momentum space, with $m = 0$

$$(\gamma^\mu p_\mu)\,\psi = 0$$

$$\left(\gamma^0 p_0 + \gamma^i p_i\right)\psi = 0$$

$$\left(\gamma^0 E - \gamma^i p^i\right)\psi = 0$$



- Using the Weyl representation

$$\begin{pmatrix} 0 & E - \sigma^i p^i \\ E + \sigma^i p^i & 0 \end{pmatrix} \begin{pmatrix} \psi_A \\ \psi_B \end{pmatrix} = 0 \Rightarrow H^\pm = \pm \mathbf{p} \cdot \sigma$$

- $\psi_A, \ \psi_B$ are eigenstates of γ_5: helicity is a good quantum number

Weyl semimetals

- Valence and conduction bands touch
- Non-degenerate band: time-reversal symmetry is broken
- One possible Hamiltonian

$$H = -\sum_{\mathbf{k}} \left[2t_x \left(\cos k_x - \cos k_0\right) + m \left(2 - \cos k_y - \cos k_z\right)\right] \sigma_x + 2t_y \sin k_y \sigma_y + 2t_z \sin k_z \sigma_z$$

- Expanding around $\mathbf{k} = \pm\left(k_0, 0, 0\right)$

$$H_{\pm} = v_x \left[p_{\pm}\right]_x \sigma_x + v_y \left[p_{\pm}\right]_y \sigma_y + v_z \left[p_{\pm}\right]_z \sigma_z$$

$$v_x = 2t_x \sin k_0, \;\; v_{y,z} = -2t_{yz}$$

- where

$$H^{\pm} = \pm \mathbf{p} \cdot \sigma$$ Weyl equation

How robust are the Weyl nodes?

- The system is 3D! Expanding around a node

$$H = \begin{pmatrix} \delta E & \psi_1 + i\psi_2 \\ \psi_1 - i\psi_2 & -\delta E \end{pmatrix}$$

$$E = \pm\sqrt{(\delta E)^2 + \psi_1^2 + \psi_2^2}$$

$$\delta E = \delta E\left(k_x, k_y, k_z\right)$$
$$\psi_1 = \psi_1\left(k_x, k_y, k_z\right)$$
$$\psi_2 = \psi_2\left(k_x, k_y, k_z\right)$$

- Solve for $E = 0$

$$\left.\begin{aligned} \delta E\left(k_x, k_y, k_z\right) &= 0 \\ \psi_1\left(k_x, k_y, k_z\right) &= 0 \\ \psi_2\left(k_x, k_y, k_z\right) &= 0 \end{aligned}\right\}$$

3 equations; 3 unknowns

Magnetic Monopoles (review)

- What is the magnetic charge of a Dirac node?
- Consider the eingestates of $H = \mathbf{k} \cdot \sigma$

$$|\psi_-(\theta,\phi)\rangle = \begin{pmatrix} \sin\frac{\theta}{2} e^{-i\phi} \\ -\cos\frac{\theta}{2} \end{pmatrix} \qquad |\psi_+(\theta,\phi)\rangle = \begin{pmatrix} \cos\frac{\theta}{2} e^{-i\phi} \\ \sin\frac{\theta}{2} \end{pmatrix}$$

- The Berry flux is $\mathbf{A} = -i \sum_{n,\mathrm{occ}} \langle u_{n,\mathbf{k}} | \nabla_{\mathbf{k}} | u_{n,\mathbf{k}} \rangle$

$$\mathbf{A} = -i \left\langle \psi_- \middle| \left(\hat{\mathbf{k}} \frac{\partial}{\partial k} + \frac{\hat{\theta}}{k} \frac{\partial}{\partial \theta} + \frac{\hat{\phi}}{k \sin\theta} \frac{\partial}{\partial \phi} \right) \middle| \psi_- \right\rangle \qquad \mathbf{B} = \nabla_{\mathbf{k}} \times \mathbf{A}$$

$$\int d\mathbf{S} \cdot \mathbf{B} = k^2 \int d\Omega \mathbf{B} \cdot \hat{\mathbf{k}} \qquad \mathbf{B}.\hat{\mathbf{k}} = \frac{\partial_\theta (k \sin\theta A_\phi) - \partial_\phi (k A_\theta)}{k^2 \sin\theta} = -\frac{1}{2k^2}$$

$$\int d\mathbf{S} \cdot \mathbf{B} = \pm 2\pi$$

Topological Response: Chiral Anomaly

- Breaks down the conservation of particles at a given Weyl point

$$H_{R/L} = \mp i\hbar v \psi^{\dagger}_{L/R} \sigma \cdot \left(\nabla - \frac{ie\mathbf{A}}{\hbar} \right) \psi_{L/R}$$

- Independent equations for R/L ?
- QFT: Cut-off violates conservation (Adler-Bell-Jackiw anomaly)

$$\frac{\partial n_{R,L}}{\partial t} = \pm \frac{e^2}{h^2} \mathbf{E} \cdot \mathbf{B}$$

- Another derivation: Starting with B in the z-direction
- → Landau Levels
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Topological Response: Chiral Anomaly

- Solving the Dirac equation with an applied magnetic field,

$$l_B = \sqrt{\frac{\hbar c}{eB}}$$

$$\frac{N_\phi}{A} = \frac{1}{2\pi l_B^2}$$

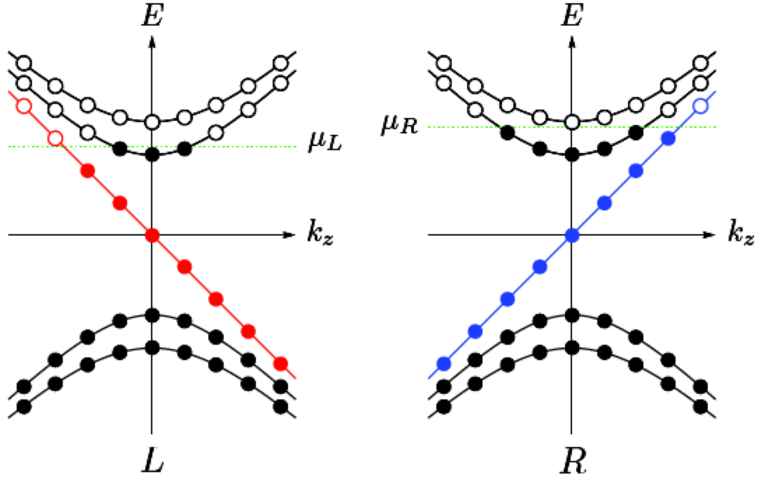
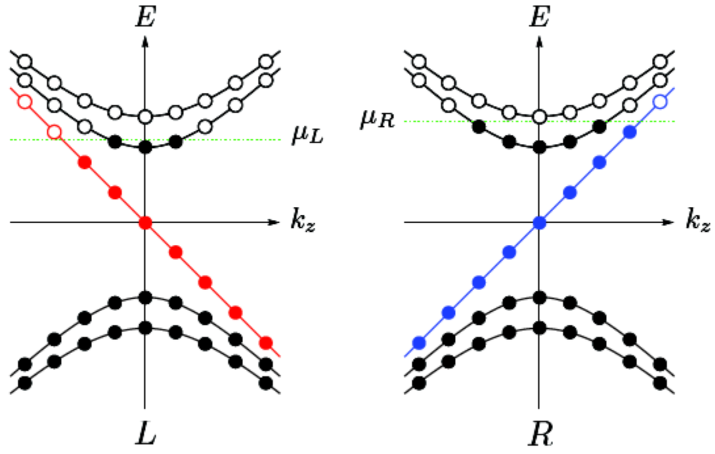


FIG. 2. **Landau levels at Weyl Nodes.** Filled (empty) circles denote occupied (empty) LLs. Each node has non-chiral LLs that disperse parabolically in the field direction (here $\hat{\mathbf{z}}$) as well as a single chiral LL that disperses according to the node chirality (red, blue). A chemical potential imbalance between the nodes leads to a net current flowing along the field, even for spatially uniform μ.

$$E_0\left(\mathbf{k}\cdot\hat{\mathbf{n}}\right) = \pm v\mathbf{k}\cdot\hat{\mathbf{n}}$$

$$E_n\left(\mathbf{k}\cdot\hat{\mathbf{n}}\right) = \hbar v_F \mathrm{sign}\left(n\right)\sqrt{\frac{2\left|n\right| eB}{\hbar c} + \left(\mathbf{k}\cdot\hat{\mathbf{n}}\right)^2}, \quad n = \pm 1, \pm 2, \ldots$$

Topological Response: Chiral Anomaly



- Effect of the electric field

$$\frac{dp_{F,L/R}}{dt} = eE_z$$

- Rate of increase/decrease of electrons

$$\frac{dN_R}{dt} = (eE_z)\left(\frac{L_z}{h}\right)\left(\frac{AB_z}{\Phi_0}\right)$$

force density of states density of modes

- Change of momentum per unit volume

$$\frac{\partial n_{R,L}}{\partial t} = \pm\frac{e^2}{h^2}\mathbf{E}\cdot\mathbf{B} \quad \Rightarrow \quad \frac{d\mathbf{p}}{dt} = \frac{e^2}{h^2}\sum_i \kappa_i \mathbf{k}_i \mathbf{E}\cdot\mathbf{B}$$

Topological Response: Anomalous Hall Effect

Using the Lorentz force

$$\frac{d\mathbf{p}}{dt} = \rho_H \mathbf{E} + \mathbf{J}_H \times \mathbf{B}$$

and the basic equations for the QHE

$$\mathbf{J}_H = \mathbf{E} \times \mathbf{G}_H \qquad \rho_H = \mathbf{B} \cdot \mathbf{G}_H$$

$$\begin{aligned} \Rightarrow \frac{d\mathbf{p}}{dt} &= (\mathbf{B} \cdot \mathbf{G}_H)\,\mathbf{E} + (\mathbf{E} \times \mathbf{G}_H) \times \mathbf{B} \\ &= (\mathbf{B} \cdot \mathbf{G}_H)\,\mathbf{E} - (\mathbf{B} \cdot \mathbf{G}_H)\,\mathbf{E} + (\mathbf{B} \cdot \mathbf{E})\,\mathbf{G}_H \\ &= (\mathbf{B} \cdot \mathbf{E})\,\mathbf{G}_H \end{aligned}$$

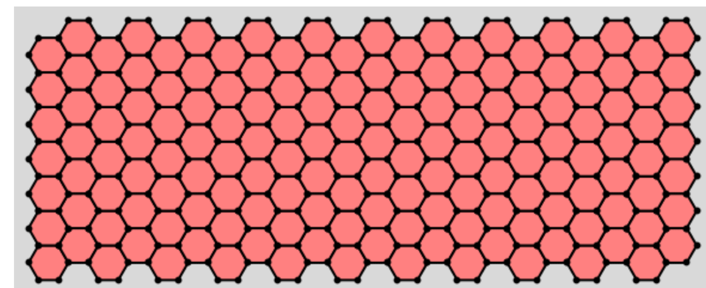
and $\frac{d\mathbf{p}}{dt} = \frac{e^2}{h^2} \sum_i \kappa_i \mathbf{k}_i \mathbf{E} \cdot \mathbf{B}$

$$\mathbf{G}_H = \frac{e^2}{2\pi\hbar} \sum_i \kappa_i \mathbf{k}_i$$

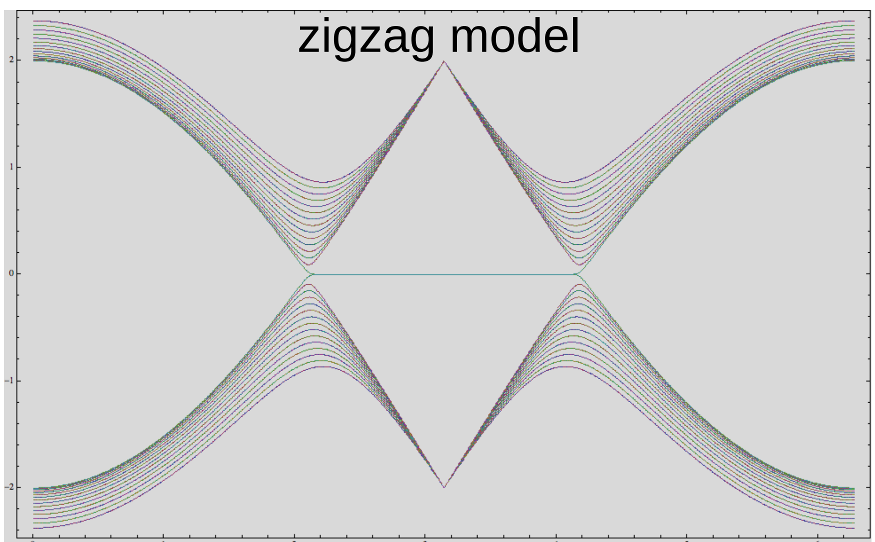
Is Graphene a Topological Semimetal?

- If just nearest-neighbor hopping is taken into account,

$$H = -t \sum_{\langle i,j \rangle, \sigma} \left(a^{\dagger}_{\sigma,i} b_{\sigma,j} + \mathrm{H.c.} \right)$$



- Sublattice symmetry: changing the sign of amplitudes on one sublattice reverses the sign
- 1D: Class AIII, edge states with zero-energy



Name	T	C	S=CT	d=1
A	0	0	0	-
AIII	0	0	1	$\mathbb{Z}$
AI	+1	0	0	-
BDI	+1	+1	1	$\mathbb{Z}$
D	0	+1	0	$\mathbb{Z}_2$
DIII	-1	+1	1	$\mathbb{Z}_2$
AII	-1	0	0	-
CII	-1	-1	1	$\mathbb{Z}$
C	0	-1	0	-
CI	+1	-1	1	-

Experimental Proposal

- Goal: distinguish anomaly-related physics from conventional metallic behavior

$$|V_{\rm nl}(x)| = V_{\rm SD} e^{-x/\ell_v}$$

$$\ell_v = \sqrt{D\tau_v}$$

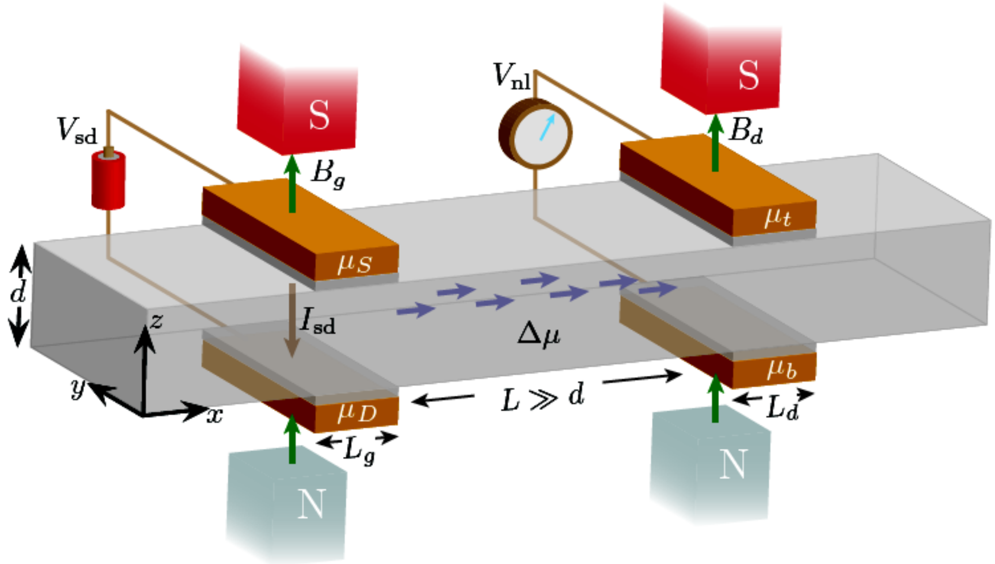


FIG. 1. **Nonlocal Transport Experiment.** A source-drain current $I_{\rm sd}$ is injected into a Weyl semimetal slab of thickness d via tunneling contacts of thickness L_g. In the presence of a local 'generation' magnetic field B_g, a valley imbalance $\Delta\mu$ is created via the chiral anomaly and diffuses a distance $L \gg d$ away. If a 'detection' field B_d is applied, the valley imbalance can be converted into a potential difference $V_{\rm nl}$ between top and bottom contacts of size L_d.